Oblique incidence for lossy dialectrics

$$\sigma_1 = 0 = \sigma_2$$

$$\alpha_1 = \alpha_2 = 0$$

$$\gamma = \alpha + j\beta = j\beta \qquad \boxed{\beta = k}$$

$\hat{n}_i$: direction of propogation

$$\boxed{\beta \hat{n}_i = \vec{k_i} = k_i \hat{k}}$$

$$\sigma_1 = 0 \Rightarrow \alpha_1 = 0 \qquad \beta_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\vec{E_{si}} = \vec{E_{is}}\, e^{-j\beta \hat{n}_i \cdot \vec{r_i}}$$

$$= \vec{E_{is}}\, e^{-j\vec{k_i} \cdot \vec{r_i}}$$

$$= \vec{E_{is}}\, e^{-jk_i \hat{k_i} \cdot \vec{r_i}}$$

$\vec{r_i}$: pt is space in medium i.

"nice relations"

$$\vec{k_i} \times \vec{E_{is}} = \omega_i \mu_i \vec{H_{is}}$$

$$\vec{H_{is}} \times \vec{k_i} = \omega_i \varepsilon_i \vec{E_{is}}$$

Properties of plane waves:

$$\vec{k_i} \cdot \vec{E_i} = \vec{k_i} \cdot \vec{H_i} = \vec{E_i} \cdot \vec{H_i}$$

$$\vec{k} = \omega_i \sqrt{\mu_i \varepsilon_i}\ \hat{k}_i$$

$$\vec{H_i} = \frac{1}{\eta_i} \hat{k}_i \times \vec{E_i}$$

$$\vec{E_i}(z,t) = \vec{E_{is}}(\vec{r}) \cos(\omega_i t - \beta \hat{n}_i \cdot \vec{r})$$

$$= \vec{E_{is}}(\vec{r}) \cos(\omega_i t - \vec{k_i} \cdot \vec{r})$$

physical feild @ $\vec{r}, t$; constant vector (assumed real)
i = incident.

$$= \vec{E_{io}}(\vec{r}) \cos(\omega_i t - k_{ix}x - k_{iy}y - k_{iz}z)$$

note: for the normal incident $k_{ix} = k_{iy} = 0$

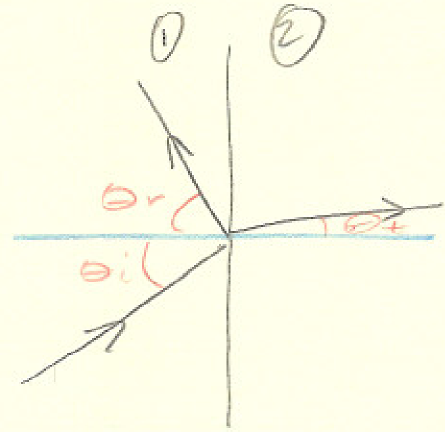
Reflected feild

$$\vec{E_r}(\vec{r},t) = \vec{E_{ro}}(\vec{r}) \cos(\omega_r t - k_{rx}x - k_{ry}y - k_{rz}z)$$

Transmitted feild

$$\vec{E_t}(\vec{r},t) = \vec{E_{to}}(\vec{r}) \cos(\omega_r t - k_{tx}x - k_{ty}y - k_{tz}z)$$

Step 1 interface @ $z=0$ in the xy plane



$$\boxed{z=0}$$

note This means no conduction on the z components.

$$\oint \vec{E}\cdot d\vec{l} = 0 \qquad \int \vec{D}\cdot d\vec{s} = 0$$

conditions on tangental components of E

no free charge on the surface of our interface.

∴ Net feild in medium 1 @ intrface

$$\vec{E_{io}}(\vec{r})\cos(w_i t - k_{ix}x - k_{iy}y)$$
$$+ \vec{E_{ro}}(\vec{r})\cos(w_i t - k_{rx}x - k_{ry}y)$$

The net feild in medium 2 @ interface

$$\vec{E}_{to}(\vec{r})\cos(\omega t - k_{tx}x - k_{ty}y)$$

$\int \vec{E}\cdot d\vec{l}$ produces relations between the 3 electric feilds.

$$\vec{E}_{io\parallel}(\vec{r})\cos(\omega_i t - k_{ix}x - k_{iy}y)$$
$$+ \vec{E}_{ro\parallel}(\vec{r})\cos(\omega_r t - k_{rx}x - k_{ry}y) =$$
$$\vec{E}_{to\parallel}(\vec{r})\cos(\omega_t t - k_{tx}x - k_{ty}y)$$

This must hold for every $\vec{r} = (x, y, 0)$

In particular, it must hold for $\vec{r} = (0,0,0)$ and for all t,

$$\vec{E}_{io}(\vec{r})\cos(\omega_i t) + \vec{E}_{ro}(\vec{r})\cos(\omega_r t)$$
$$= \vec{E}_{to}(\vec{r})\cos(\omega_t t)$$

The freq must be the same throughout

$$\omega_i = \omega_t = \omega_r = \omega$$

Pick any t and $x = 0$ $(t=0)$

$$E_{io}\cos(-k_{iy}y) + E_{ro}\cos(-k_{ry}y)$$

$$= E_{to} \cos(-k_{ty} y)$$

$\therefore$ for every y

$$k_{iy} = k_{ty} = k_{ry}$$

And similiarly

$$k_{ix} = k_{iy} = k_{iz}.$$